Vectors

1. SCALAR QUANTITY & VECTOR QUANTITY

Scalar Quantity :

A quantity which has only magnitude and not related to any direction is called a scalar quantity.

For example, Mass, Length, Time, Temperature, Area, Volume, Speed, Density, Work, current etc

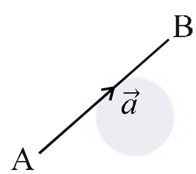
Vector Quantity :

A quantity which has magnitude and also a direction in space and which obeys triangle law of addition of vectors, is called a vector quantity.

For example, Displacement, Velocity, Acceleration, Force, Torque, etc.

2. REPRESENTATION OF VECTORS :

Vectors are represented by directed line segments. A vector $\vec{a}$ is represented by the directed line segment $\overrightarrow{AB}$. The magnitude of vector $\vec{a}$ is equal is equal to AB and the direction of vector $\vec{a}$ is along the line form A to B.



3. TYPE OF VECTORS :

(i) Null vector or zero vector : If the initial and terminal points of a vector coincide, then it is called a zero vector. It is denoted by $\vec{0}$. Its magnitude is zero and direction is indeterminate.

(ii) Unit vector : A vector whose magnitude is of unit length along any vector $\vec{a}$ is called a unit vector in the direction of $\vec{a}$ and is denoted by $\hat{a}$, where $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

Note : (a) $|\hat{a}| = 1$.

(b) Two unit vectors may not be equal unless they have the same direction.

(c) Unit vectors parallel to x-axis, y-axis and z-axis are denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively.

(iii) Reciprocal vector : A vector whose direction is same as that of a given vector $\vec{a}$ but its magnitude is the reciprocal of the magnitude of the given vector $\vec{a}$ is called the reciprocal of $\vec{a}$ and is denoted by a^{-1}.

(iv) Equal vector : Two non-zero vectors are said to be equal vectors if their magnitudes are equal and directions are same i.e. they act parallel to each other in the same direction.

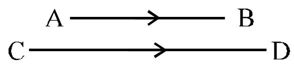
(v) Negative vector : The negative of a vector is defined as the vector having the same magnitude but opposite direction.

For example, if $\vec{a} = \overrightarrow{PQ}$, then the negative of $\vec{a}$ is the vector $\overrightarrow{QP}$ and is denoted as $-\vec{a}$.

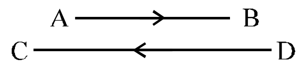
(vi) Collinear vector : Two or more non-zero vectors are said to be collinear vectors if these are parallel to the same line.

(vii) Like and unlike vector : Collinear vectors having the same direction are known as like vectors while those having opposite direction are known as unlike vectors.

For example, the vectors given by figure (a) are like and given by figure (b) are unlike vectors.



(a)



(b)

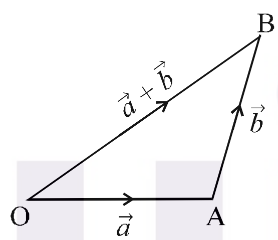
(viii) Coplanar vector : Two or more non-zero vectors are said to be coplanar vectors if these are parallel to the same plane.

(ix) Localised vector and free vector : A vector drawn parallel to a given vector through a specified point as the initial point, is known as a localised vector. If the initial point of a vector is not specified it is said to be a free vector.

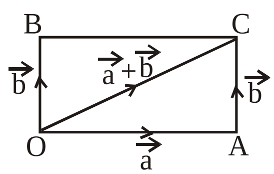
(x) Position vector : Let O be the origin and let A be a point such that $\overrightarrow{OA} = \vec{a}$ then, we say that the position vector of A is $\vec{a}$.

4. ADDITION OF VECTORS :

Let $\vec{a}$ and $\vec{b}$ be any two vectors. From the terminal point of $\vec{a}$, vector $\vec{b}$ is drawn. Then, the vector from the initial point O of $\vec{a}$ to the terminal point B of $\vec{b}$ is called the sum of vectors $\vec{a}$ and $\vec{b}$ and is denoted by $\vec{a} + \vec{b}$. This is called the triangle law of addition of vectors.



The vectors are also added by using the following method. Let $\vec{a}$ and $\vec{b}$ be any two vectors. From the initial point of $\vec{a}$, vector $\vec{b}$ is drawn. let O be their common initial point. If A and B be respectively the terminal points of $\vec{a}$ and $\vec{b}$, then parallelogram OACB is completed with OA and OB as adjacent sides. The vector $\overrightarrow{OC}$ is defined as the sum of $\vec{a}$ and $\vec{b}$. This is called the parallelogram law of addition of vectors.



(a) Properties of Vector Addition :

(i) Vector addition is commutative, i.e. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$.

(ii) Vector addition is associative, i.e. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$

(iii) $\vec{O} + \vec{a} = \vec{a} + \vec{O} = \vec{a}$ .So, the $\vec{a}$ is additive identity.

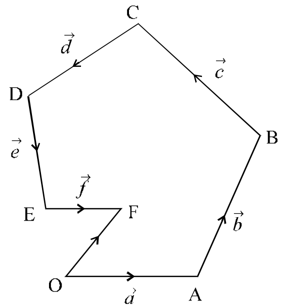
(iv) $\vec{a} + (-\vec{a}) = \vec{O} = (-\vec{a}) + \vec{a}$. So, the additive inverse of $\vec{a}$ is $-\vec{a}$.

(b) Polygon Law of Addition of Vectors

To find the sum of any number of vectors we represent the vectors by directed line segment with the terminal point of the previous vector as the initial point of the next vector. Then the line segment joining the initial point of the first vector to the terminal point of the last vector will represent the sum of the vectors :

Thus if, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{AB} = \vec{b}$, $\overrightarrow{BC} = \vec{c}$, $\overrightarrow{CD} = \vec{d}$, $\overrightarrow{DE} = \vec{e}$ and $\overrightarrow{EF} = \vec{f}$ then

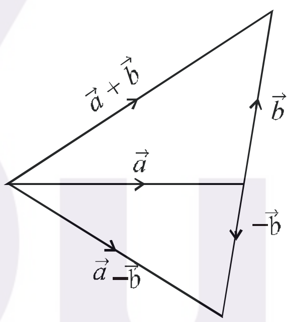
$$\vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e} + \vec{f} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} = \overrightarrow{OF}$$



If the terminal point F of the last vector coincide with initial point of the first vector then $\vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e} + \vec{f} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EO} = \vec{O}$, i.e. the sum of vectors is zero or null vector in this case.

5. DIFFERENCE OF VECTORS :

If $\vec{a}$ and $\vec{b}$ be any two vectors, then their difference $\vec{a} - \vec{b}$ is defined as $\vec{a} + (-\vec{b})$.



6. MULTIPLICATION OF A VECTOR BY A SCALAR :

If $\vec{a}$ be any vector and m any scalar, then the multiplication of $\vec{a}$ by m is defined as a vector having magnitude $|m||\vec{a}|$ and direction same as of $\vec{a}$, if m is positive and reversed if m is negative. The product of $\vec{a}$ and m is denoted by m $\vec{a}$. If m = 0, then m$\vec{a}$ is the zero vector.

For example, if $\vec{a} = \overrightarrow{AB}$ then $|2\vec{a}| = |2||\vec{a}| = 2|\vec{a}|$ and direction same as that of $\vec{a}$.

The magnitude of the vectors $-3\vec{a}| = 3|\vec{a}|$ and direction opposite as that of $\vec{a}$.

7. IMPORTANT PROPERTIES AND FORMULAE :

1. (a) Triangle law of vector addition $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

(b) Parallelogram law of vector addition : If ABCD is a parallelogram, then $\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$

(c) If $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ then

$\vec{r}_1 + \vec{r}_2 = (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j} + (z_1 + z_2)\hat{k}$ and $\vec{r}_1 = \vec{r}_2 \Leftrightarrow x_1 = x_2,\ y_1 = y_2,\ z_1 = z_2$

2. (a) $\vec{a}$ and $\vec{b}$ are parallel if and only if $\vec{a} = m\vec{b}$ for same non-zero scalar m.

(b) $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$ or $\vec{a} = |\vec{a}|\,\hat{a}$

(c) Associative law : $m(n\vec{a}) = (mn)\,\vec{a} = n\,(m\vec{a})$

(d) Distributive laws : $(m+n)\,\vec{a} = m\vec{a} + n\vec{a}$ and $n\,(\vec{a}+\vec{b}) = n\vec{a} + n\vec{b}$

(e) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $m\vec{r} = mx\hat{i} + my\hat{j} + mz\hat{k}$.

(f) $\vec{r}, \vec{a}, \vec{b}$ are coplaner if and only $\vec{r} = x\vec{a} + y\vec{b}$ for some scalars x and y.

3. (a) If the position vectors of the points A and B be $\vec{a}$ and $\vec{b}$ then,

(i) The position vectors of the points dividing the line AB in the ratio m : n internally and externally are $\dfrac{m\vec{b}+n\vec{a}}{m+n}$ and $\dfrac{m\vec{b}-n\vec{a}}{m-n}$.

(ii) Position vector of the middle point of AB is given by $\dfrac{1}{2}(\vec{a}+\vec{b})$.

(iii) $\overrightarrow{AB} = \vec{b} - \vec{a}$

(b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

(c) The points A, B, C will be collinear if and only if $\overrightarrow{AB} = m\,\overrightarrow{AC}$, for some non-zero scalar m.

(d) Given vectors $x_1\vec{a} + y_1\vec{b} + z_1\vec{c},\ x_2\vec{a} + y_2\vec{b} + z_2\vec{c},\ x_3\vec{a} + y_3\vec{b} + z_3\vec{c}$, where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors, will be coplanar if and only if $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$

(e) Method to prove four points to be coplanar : To prove that the four points A, B, C and D are coplanar. Find the vector $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ and then prove them to be coplanar by the method of coplanarity i.e. one of them is a linear combination of the other two.

(f) $|\vec{a}| - |\vec{b}| \le |\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$

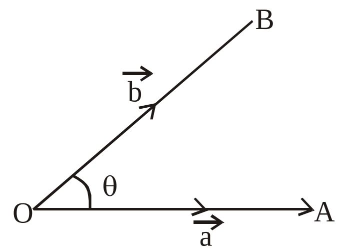
$|\vec{a}-\vec{b}| \ge |\vec{a}| - |\vec{b}|$

4. If $\vec{a}$ and $\vec{b}$ are non-zero and non - collinear vectors such that $x\vec{a} + y\vec{b} = \vec{0} \Rightarrow x = 0, y = 0$

5. If $\vec{a}, \vec{b}, \vec{c}$ are non-zero and non coplanar vectors such that $x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$

$\Rightarrow x = 0, y = 0, z = 0$

14. SCALAR PRODUCT OR DOT PRODUCT :



(a) $\vec{a} \cdot \vec{b} = ab\cos\theta$, where $0 \le \theta \le \pi$

(b) $\vec{a} \cdot \vec{b} = a$ (Projection of $\vec{b}$ along $\vec{a}$)

(c) Projection of $\vec{b}$ along $\vec{a} = \dfrac{\vec{b}.\vec{a}}{|\vec{a}|}$

(d) If $\vec{a}\,.\,\vec{b} = 0 \Rightarrow \vec{a} = 0$ or $\vec{b} = 0$ or $\vec{a}_1 \perp \vec{b}$

(e) Component of a vector $\vec{r}$ in the direction of $\vec{a}$ and perpendicular to $\vec{a}$ are $\left(\dfrac{\vec{r}.\vec{a}}{|\vec{a}|^2}\right)\vec{a}$ and $\vec{r} - \left(\dfrac{(\vec{r}.\vec{a})}{|\vec{a}|^2}\right)\vec{a}$ respectively.

(f) $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|\,|\vec{b}|}$

(g) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

$\hat{i}.\hat{j} = \hat{j}.\hat{i} = \hat{j}.\hat{k} = \hat{k}.\hat{j} = \hat{k}.\hat{i} = \hat{i}.\hat{k} = 0$

(h) If $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ i.e. if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

(i) $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

(ii) $\cos\theta = \dfrac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\,\sqrt{b_1^2 + b_2^2 + b_3^2}}$

(iii) $\vec{a}$ and $\vec{b}$ will be perpendicular if and only if $a_1b_1 + a_2b_2 + a_3b_3 = 0$

(iv) $\vec{a}$ and $\vec{b}$ will be parallel if and only if $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3}$

9. LINEARLY DEPENDENT VECTORS :

A set of vectors $\vec{a}_1, \vec{a}_2 \ldots\ldots \vec{a}_n$ is said t be linearly dependent if there exists n scalers x_1, x_2, x_3, ... x_n (not all zero) such that $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 + \ldots\ldots + x_n\vec{a}_n = \vec{0}$

10. LINEARLY INDEPENDENT VECTORS :

A set of n vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots\ldots, \vec{a}_n$ is said to be linearly independent if

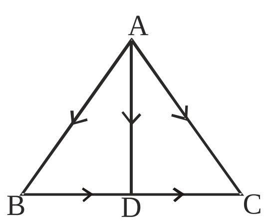
$$x_1\vec{a}_1 + x_2\vec{a}_2 + \ldots\ldots + x_n\vec{a}_n = 0 \Rightarrow x_1 = x_2 = \ldots\ldots x_n = 0$$

Note :

1. Two vectors are linearly depend if they are parallel.
2. Two vectors are linearly independent if they are non-parallel.

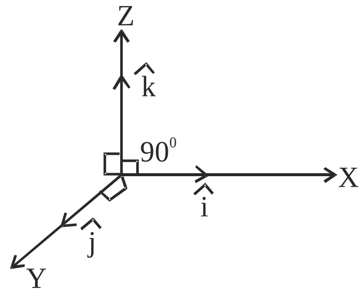
11. Direction cosines of $\vec{r} = x\hat{i} + y\hat{j} + 2\hat{k}$ are $\dfrac{x}{\sqrt{\sum x^2}}, \dfrac{y}{\sqrt{\sum x^2}}, \dfrac{z}{\sqrt{\sum x^2}}$ where $\sum x^2 = x^2 + y^2 + z^2$

12.



If D be the mid-point of BC, then $\vec{AB} + \vec{AC} = 2\vec{AD}$

13. A triad of unit vectors



14. Linear combination of vectors

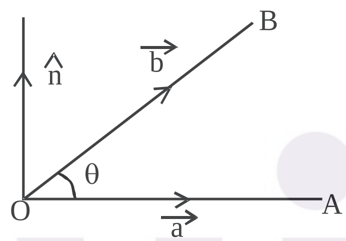
If $x_1, x_2, ..., x_n$ be scalers and $\vec{a}_1, \vec{a}_2, \vec{a}_3, ..., \vec{a}_n$ be vectors such that

$$\vec{r} = x_1\vec{a}_1, x_2\vec{a}_2, x_3\vec{a}_3, ... + x_n\vec{a}_n \text{, then}$$

$\vec{r}$ is called the linear combination of vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3, ..., \vec{a}_n$.

15. VECTOR OR CROSS PRODUCT OF TWO VECTORS :

(a) The product of vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} \times \vec{b}$.



$\vec{a} \times \vec{b} = (|\vec{a}|\,|\vec{b}|\sin\theta)\,\hat{n}$, where $\hat{n}$ is a vector $\perp^r$ to $\vec{a}$ and $\vec{b}$ both and $0 \le \theta \le \pi$

(b) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

(c) If $\vec{a} = \vec{b}$ or if $\vec{a}$ is parallel to $\vec{b}$, then $\sin\theta = 0$ and so $\vec{a} \times \vec{b} = 0$

(d) Distributive laws : $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$ and $(\vec{b} + \vec{c}) \times \vec{a} = \vec{b} \times \vec{a} + \vec{c} \times \vec{a}$

(e) The vector product of a vector $\vec{a}$ with itself is a null vector, i.e. $\vec{a} \times \vec{a} = \vec{0}$

(f) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

(i) $\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2)\,\hat{i} + (a_3b_1 - a_1b_3)\,\hat{j} + (a_1b_2 - a_2b_1)\,\hat{k}$

i.e $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(ii) $\sin^2\theta = \dfrac{(a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 + (a_1b_2 - a_2b_1)^2}{(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)}$

(g) If two vectors $\vec{a}$ and $\vec{b}$ are parallel, then $\theta = 0$ or π i.e. $\sin\theta = 0$ in both cases.

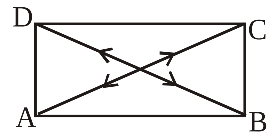
$\because \quad (a_1b_2 - a_2b_1)^2 + (a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 = 0$

$\Rightarrow \quad a_1b_2 - a_2b_1 = 0,\ a_2b_3 - a_3b_2 = 0,\ a_3b_1 - a_1b_3 = 0$

$\Rightarrow \quad \dfrac{a_1}{b_1} = \dfrac{a_2}{b_2},\ \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3},\ \dfrac{a_3}{b_3} = \dfrac{a_1}{b_1} \qquad \Rightarrow \quad \dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3}$

Thus, two vectors $\vec{a}$ and $\vec{b}$ are parallel if their corresponding components are proportional.

(h) Area of the parallelogram $ABCD = |\overrightarrow{AB} \times \overrightarrow{AD}|$ or $\frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{BD}|$



(i) Area of the triangle $ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

(j) $\vec{a} \times \vec{b}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$

(k) The unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is given by

$$\hat{n} = \pm \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$$

(l) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$

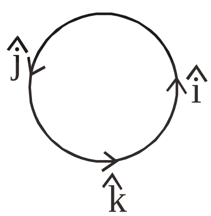
$\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ but

$\hat{i} \times \hat{k} = -\hat{j}$

$\hat{k} \times \hat{j} = -i$

$\hat{j} \times \hat{i} = \hat{k}$



16. SCALAR TRIPLE PRODUCT :

(a) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, then

$$(\vec{a} \times \vec{b}).\vec{c} = [\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(b) [a b c] = volume of the parallelopiped whose coterminous edges are formed by $\vec{a}, \vec{b}, \vec{c}$.

(c) [a b c] = [b c a] = [c a b] but [a b c] = – [a c b] etc.

i.e. change of any two vector in scalar triple product changes the sign of the scalar triple product.

(d) If any two of the vectors $\vec{a}, \vec{b}, \vec{c}$ are equal, then [a b c] = 0

(e) The position of dots and cross in a scalar triple product can be interchanged. Hence $(\vec{a} \times \vec{b}).\vec{c} = \vec{a}.(\vec{b} \times \vec{c})$

(f) The value of a scalar triple product is zero, if two of its vectors are parallel.

(g) Vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if and only if $[\vec{a} \ \vec{b} \ \vec{c}] = 0$

(h) Four points A, B, C, D with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively are coplanar if and only if $[\overrightarrow{AB} \ \overrightarrow{AC} \ \overrightarrow{AD}] = 0$ i.e. if and only if $[\vec{b} - \vec{a}, \ \vec{c} - \vec{a}, \ \vec{d} - \vec{a}] = 0$

(i) Volume of a tetrahedron with three coterminous edges $\vec{a}, \vec{b}, \vec{c} = \frac{1}{6}\left|[\vec{a} \ \vec{b} \ \vec{c}]\right|$

17. VECTOR TRIPLE PRODUCT :

If $\vec{a}, \vec{b}, \vec{c}$ be any three vectors, then $(\vec{a} \times \vec{b}) \times \vec{c}$ and $\vec{a} \times (\vec{b} \times \vec{c})$ are known as vector triple product.

(a) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\,\vec{b} - (\vec{a}.\vec{b}).\vec{c}$

$(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}.\vec{c})\,\vec{b} - (\vec{b}.\vec{c}).\,\vec{a}$

(b) $\vec{a} \times (\vec{b} \times \vec{c})$ is a vector in the plane of vectors $\vec{b}$ and $\vec{c}$

(c) The vector triple product is not commutative i.e

$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$

18. SCALAR PRODUCT OF FOUR VECTORS

The vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ combined is the form of $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d})$ is called the scalar product of four vectors.

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a}.\vec{c} & \vec{a}.\vec{d} \\ \vec{b}.\vec{c} & \vec{b}.\vec{d} \end{vmatrix} = (\vec{a}.\vec{c})\,(\vec{b}.\vec{d}) - (\vec{a}.\vec{d})\,(\vec{b}.\vec{c})$$

19. VECTOR PRODUCT OF FOUR VECTORS :

If four vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are combined in the form $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$ is called the vector product of four vectors. Here $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\,\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d} = [\vec{a}\ \vec{c}\ \vec{d}]\vec{b} - [\vec{b}\ \vec{c}\ \vec{d}]\vec{a}$

20. RECIPROCAL SYSTEM OF VECTORS :

(a) If $\vec{a}, \vec{b}, \vec{c}$ be any three non-coplanar vectors such that $[\vec{a}\ \vec{b}\ \vec{c}] \neq 0$, then the three vectors $\vec{a}', \vec{b}', \vec{c}'$ defined by the equations $\vec{a}' = \dfrac{\vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{b}' = \dfrac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{c}' = \dfrac{\vec{a} \times \vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ are called the reciprocal system of vectors to the given vectors $\vec{a}, \vec{b}, \vec{c}$.

(b) Properties :

(i) $\vec{a}.\vec{a}' = \vec{b}.\vec{b}' = \vec{c}.\vec{c}' = 1$

(ii) The scalar product of any vector of one system with a vector of the other system which does not correspond to it, is zero, i.e. $\vec{a}.\vec{b}' = \vec{a}.\vec{c}' = \vec{b}.\vec{a}' = \vec{b}.\vec{c}' = \vec{c}.\vec{a}' = \vec{c}.\vec{b}' = 0$

(iii) $[\vec{a}\ \vec{b}\ \vec{c}]\ [\vec{a}'\ \vec{b}'\ \vec{c}'] = 1$

(iv) $\hat{i}' = \hat{i},\ \hat{j}' = \hat{j},\ \hat{k}' = \hat{k}$

(v) If $\{\vec{a}', \vec{b}', \vec{c}'\}$ is reciprocal system of $\{\vec{a}, \vec{b}, \vec{c}\}$ and $\vec{r}$ is any vector, then

$\vec{r} = (\vec{r}.\vec{a})\,\vec{a}' + (\vec{r}.\vec{b})\,\vec{b}' + (\vec{r}.\vec{c})\,\vec{c}'$

$\vec{r} = (\vec{r}.\vec{a}')\,\vec{a} + (\vec{r}.\vec{b}')\,\vec{b} + (\vec{r}.\vec{c}')\,\vec{c}$

21. APPLICATION OF VECTOR IN MECHANICS

(a) Work done by a force $\vec{F}$ = $\vec{F}.\vec{d}$ where $\vec{d}$ is displacement.

(b) Moment of a force $\vec{F}$ about a point $O = \overrightarrow{OP} \times \vec{F}$, where P is any point on the line of action of the force $\vec{F}$.

(c) Moment of the couple $= \vec{r} \times \vec{F}$

22. APPLICATION OF VECTOR IN GEOMETRY :

(a) Vector equation of a straight line passing through a point $\vec{a}$ and parallel to $\vec{b}$ is $\vec{r} = \vec{a} + t\vec{b}$ where t is an arbitrary constant.

(b) Vector equation of a straight line passing through two points $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + t(\vec{b} - \vec{a})$.

(c) Vector equation of a plane passing through a point $\vec{a}$ and parallel to two given vectors $\vec{b}$ and $\vec{c}$ is $\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$, where t and s are arbitrary constants.

or $[\vec{r}\ \vec{b}\ \vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}]$

(d) Vector equation of a plane passing through the points $\vec{a}, \vec{b}, \vec{c}$ is $\vec{r} = (1 - s - t)\vec{a} + s\vec{b} + t\vec{c}$.

or $\vec{r}.(\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}) = [\vec{a}\ \vec{b}\ \vec{c}]$

(e) Vector equation of a plane passing through the point $\vec{a}$ and perpendicular to $\vec{n}$ is $\vec{r}.\vec{n} = \vec{a}.\vec{n}$.

Note : Perpendicular distance of the plane from the origin $= \dfrac{\vec{a}.\vec{n}}{|\vec{n}|}$

(f) Perpendicular distance of a point $P(\vec{r})$ from a line passing through $\vec{a}$ and parallel to $\vec{b}$ is given by

$$PM = \frac{|(\vec{r} - \vec{a}) \times \vec{b}|}{|\vec{b}|} = \left\{ (\vec{r} - \vec{a})^2 - \left(\frac{(\vec{r} - \vec{a}).\vec{b}}{|\vec{b}|} \right)^2 \right\}^{1/2}$$

(g) Perpendicular distance of a point $P(\vec{r})$ from a plane passing through $\vec{a}$ and parallel to $\vec{b}$ and $\vec{c}$ is given by

$$PM = \frac{(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|}$$

(h) Perpendicular distance of a point $P(\vec{r})$ from a plane passing through the points $\vec{a}$, $\vec{b}$ and $\vec{c}$ is given by

$$PM = \frac{(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b})}{|\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}|}$$